Probar usando las propiedades de los determinantes.

$$\begin{vmatrix}1 & x & x(x^2-1)\\ 1 & y & y(y^2-1)\\ 1 & z & z(z^2-1)\end{vmatrix} = \begin{vmatrix}y+z & x & yz\\ x+z & y & xz\\ x+y & z & xy\end{vmatrix}$$

Solución.

$$\begin{vmatrix}y+z & x & yz\\ x+z & y & xz\\ x+y & z & xy\end{vmatrix} = \left|C_{12}(1)\cdot\begin{pmatrix}y+z & x & yz\\ x+z & y & xz\\ x+y & z & xy\end{pmatrix}\right| = \begin{vmatrix}x+y+z & x & yz\\ x+y+z & y & xz\\ x+y+z & z & xy\end{vmatrix}$$

$$= (x+y+z)\cdot\begin{vmatrix}1 & x & yz\\ 1 & y & xz\\ 1 & z & xy\end{vmatrix} = \begin{vmatrix}1 & x(x+y+z) & yz\\ 1 & y(x+y+z) & xz\\ 1 & z(x+y+z) & xy\end{vmatrix}$$

$$= \begin{vmatrix}1 & x^2+xy+xz & yz\\ 1 & y^2+xy+yz & xz\\ 1 & z^2+xz+yz & xy\end{vmatrix} = \left|C_{23}(1)\cdot\begin{pmatrix}1 & x^2+xy+xz & yz\\ 1 & y^2+xy+yz & xz\\ 1 & z^2+xz+yz & xy\end{pmatrix}\right|$$

$$= \begin{vmatrix}1 & x^2+xy+xz+yz & yz\\ 1 & y^2+xy+xz+yz & xz\\ 1 & z^2+xy+xz+yz & xy\end{vmatrix} = \left|C_{21}(-xy-xz-yz)\cdot\begin{pmatrix}1 & x^2+xy+xz+yz & yz\\ 1 & y^2+xy+xz+yz & xz\\ 1 & z^2+xy+xz+yz & xy\end{pmatrix}\right|$$

$$= \begin{vmatrix}1 & x^2 & yz\\ 1 & y^2 & xz\\ 1 & z^2 & xy\end{vmatrix} = xyz\cdot\begin{vmatrix}1 & x^2 & \frac{1}{x}\\ 1 & y^2 & \frac{1}{y}\\ 1 & z^2 & \frac{1}{z}\end{vmatrix}$$

$$= yz\cdot\begin{vmatrix}x & x^3 & 1\\ 1 & y^2 & \frac{1}{y}\\ 1 & z^2 & \frac{1}{z}\end{vmatrix} = z\cdot\begin{vmatrix}x & x^3 & 1\\ y & y^3 & 1\\ 1 & z^2 & \frac{1}{z}\end{vmatrix}$$

$$= \begin{vmatrix}x & x^3 & 1\\ y & y^3 & 1\\ z & z^3 & 1\end{vmatrix} = \left|C_{21}(-1)\cdot\begin{pmatrix}x & x^3 & 1\\ y & y^3 & 1\\ z & z^3 & 1\end{pmatrix}\right|$$

$$= \begin{vmatrix}x & x^3-x & 1\\ y & y^3-y & 1\\ z & z^3-z & 1\end{vmatrix} = \begin{vmatrix}x & x(x^2-1) & 1\\ y & y(y^2-1) & 1\\ z & z(z^2-1) & 1\end{vmatrix}$$

$$= -\left|C_{23}\cdot\begin{pmatrix}x & x(x^2-1) & 1\\ y & y(y^2-1) & 1\\ z & z(z^2-1) & 1\end{pmatrix}\right| = -\begin{vmatrix}x & 1 & x(x^2-1)\\ y & 1 & y(y^2-1)\\ z & 1 & z(z^2-1)\end{vmatrix}$$

$$= \left|C_{12}\cdot\begin{pmatrix}x & 1 & x(x^2-1)\\ y & 1 & y(y^2-1)\\ z & 1 & z(z^2-1)\end{pmatrix}\right| = \begin{vmatrix}1 & x & x(x^2-1)\\ 1 & y & y(y^2-1)\\ 1 & z & z(z^2-1)\end{vmatrix}$$

$$\therefore \text{ Queda demostrado } \begin{vmatrix}1 & x & x(x^2-1)\\ 1 & y & y(y^2-1)\\ 1 & z & z(z^2-1)\end{vmatrix} = \begin{vmatrix}y+z & x & yz\\ x+z & y & xz\\ x+y & z & xy\end{vmatrix}$$

1)

$A^2 = -I$

$(A^2)^2 = (-I)^2 = -I$

$\to A^{10} = A^8 \cdot A$

$= -I \cdot A$

$= -A$

$\therefore adj(A^{10}) = adj(-A) = adj(-1 \cdot A)$

$= adj(A)$

2)

I) $\exists A = B^{-1} D B B^{-1}$

$A = B^{-1} D$

$\to BA = D$

$\therefore |I - \lambda A| = |I - \lambda D|$ (Falso)

$|I - \lambda A| = |I - \lambda BA|$

o la igualdad depende del valor de B

II)

$A^2 = I \to A = A^{-1}$

$\therefore \det A = \dfrac{1}{\det A}$ (Falso)

$\det^2 A = 1$ $\to |A| = \{-1, 1\}$

$\det A = \pm 1$

III)

$|A| = 3$

$(adj A)^T = (A^{-1} \det A)^T$ $\to |adj((adj A)^T)| = |(3(A^T)^{-1})^{-1} \det(3(A^T)^{-1})|$

$= (3A^{-1})^T$

$= 3 (A^T)^{-1}$

$= |\frac{1}{3} A^T \, 27 \, |(A^T)^{-1}||$

$= |9 A^T \frac{1}{|A^T|}|$

$= 27 |A^T|$

$= 81 \geq 81$ (Verdadero)

3)

$= 81 \geq 81$ (Verdadero)

3)

$E_{ij}(1) E_{ij}(-1) E_{ij}(1) E_{ij}(-1) = E_{ij}$

$E_{ij}$: Matriz elemental del tipo $F_{ij(1)} I$

$E_{ij}$: Matriz elemental del tipo: $F_{ij(-1)} I$

$\Rightarrow = F_{ij(1)} I F_{ij(-1)} I F_{ij(1)} I F_{ij(-1)} I$ $\to I \neq E_{ij}$: Matriz elemental del tipo: $F_{ij} I$

$= F_{ij(1)} F_{ij(-1)} F_{ij(1)} F_{ij(-1)} I$

$= I$

22)

$$A^{-1} = F_{12}F_{23}(-1)F_2(1/3)F_{13}(2)\cdot I$$
$$\rightarrow A = F_{13}(-2)F_2(3)F_{23}(1)F_{12}\cdot I$$

$$B=\begin{bmatrix}1&1&1&1\\2&a&2&-4\\1&2&b&5\\3&a+1&3&b\end{bmatrix}$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\xrightarrow{f_{12}}\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\xrightarrow{f_{23}(1)}\begin{bmatrix}0&1&0&0\\1&0&1&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\xrightarrow{f_2(3)}\begin{bmatrix}0&1&0&0\\3&0&3&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\xrightarrow{f_{13}(-2)}\begin{bmatrix}0&1&-2&0\\3&0&3&0\\0&0&1&0\\0&0&0&1\end{bmatrix}=A$$

$$M=A\times B=\begin{bmatrix}0&1&-2&0\\3&0&3&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&1&1&1\\2&a&2&-4\\1&2&b&5\\3&a+1&3&b\end{bmatrix}=\begin{bmatrix}0&a-4&2-2b&-14\\6&9&3+3b&18\\1&2&b&5\\3&a+1&3&b\end{bmatrix}$$

M invertible si $M\,F_{xx}=I$

$$M\xrightarrow[f_{14}(-1)]{f_{23}(-3)}\begin{bmatrix}-3&-5&(-1-2b)&(-14-b)\\3&3&3&3\\1&2&b&5\\3&a+1&3&b\end{bmatrix}\xrightarrow{f_{13}}\begin{bmatrix}1&2&b&5\\3&3&3&3\\-3&-5&(-1-2b)&(-14-b)\\3&a+1&3&b\end{bmatrix}\xrightarrow[f_{31}(3)\;f_{41}(-3)]{f_{21}(-3)}\begin{bmatrix}1&2&b&5\\0&-3&3-3b&-12\\0&1&b-1&1-b\\0&a-5&3-b&b-15\end{bmatrix}$$

$$\xrightarrow{f_2(-1/3)}\begin{bmatrix}1&2&b&5\\0&1&b-1&4\\0&1&b-1&1-b\\0&a-5&3-b&b-15\end{bmatrix}\xrightarrow[f_{32}(-1)\;f_{42}(-a+5)]{f_{12}(-2)}\begin{bmatrix}1&0&2-b&-3\\0&1&b-1&4\\0&0&0&-3-b\\0&0&(4b-ab-2+a)&(b-4a+5)\end{bmatrix}\xrightarrow{f_{43}}\begin{bmatrix}1&0&2-b&-3\\0&1&b-1&4\\0&0&(4b-ab-2+a)&(b-4a+5)\\0&0&0&-3-b\end{bmatrix}$$

$$\xrightarrow{f_3\left(\frac{1}{4b-ab-2+a}\right)}\begin{bmatrix}1&0&2-b&-3\\0&1&b-1&4\\0&0&1&A\\0&0&0&-3-b\end{bmatrix}\xrightarrow[f_{23}(1-b)]{f_{13}(b-2)}\begin{bmatrix}1&0&0&-3+A(b-2)\\0&1&0&4+A(1-b)\\0&0&1&A\\0&0&0&-3-b\end{bmatrix}\xrightarrow{f_4\left(\frac{1}{-3-b}\right)}\begin{bmatrix}1&0&0&-3+A(b-2)\\0&1&0&4+A(1-b)\\0&0&1&A\\0&0&0&1\end{bmatrix}$$

$$\xrightarrow[f_{24}(-4-A(1-b))\;f_{34}(-A)]{f_{14}(3-A(b-2))}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix};$$

Con $A=\dfrac{b-4a+5}{4b-ab-2+a}$, Condiciones para que M invertible:

$4b-ab-2+a\neq 0 \rightarrow b\neq \dfrac{2-a}{4-a}$; $\rightarrow a\neq 4$

$3-b\neq 0 \rightarrow b\neq 3$

1-

$A^2 = -I$

$(A^2)^3 = (-I)^3 = -I$

$\Rightarrow A^{10} = A^9 \cdot A = -I \cdot A$
$= -A$

$\therefore adj(A^{10}) = adj(-A) = (-1)^{-1}(A)^{-1} \det(-1 \cdot A)$

$= \cancel{-1}(A)^{-1} \det(A)(-1)^3$

$= adj(A)$

2-

$\neq A = B^{-1} D B B^{-1}$

$A = B^{-1} D$

$\rightarrow BA = D$

II- (False)

$A^2 = I \rightarrow A = A^{-1}$

$\Rightarrow \det A = \dfrac{1}{\det A}$

$\det|A|^2 = 1$

$\therefore \det|A| = \pm 1 ; |A| = \{1, -1\}$

21) Sean las matrices

$$A = \begin{bmatrix} a & 2 & 0 & 0 & 0 \\ a & 3 & 2 & 0 & 0 \\ a & b & b & 2 & 0 \\ 0 & 0 & b & 3 & 2 \\ 0 & 0 & 0 & a & 2 \end{bmatrix} \text{ y } B = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

Para qué valores de a y b al rango AB será 5, 4, 3, 2 ó 1.

rango de $A \cdot B = \min\{A, B\}$

$$A = \begin{pmatrix} a & 2 & 0 & 0 & 0 \\ a & 3 & 2 & 0 & 0 \\ a & b & b & 2 & 0 \\ 0 & 0 & b & 3 & 2 \\ 0 & 0 & 0 & a & 2 \end{pmatrix} \underset{f_{31}(-1)}{\overset{f_{21}(-1)}{\sim}} \begin{pmatrix} a & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & b-2 & b & 2 & 0 \\ 0 & 0 & b & 3 & 2 \\ 0 & 0 & 0 & a & 2 \end{pmatrix} \overset{f_{32}(-(b-2))}{\sim} \begin{pmatrix} a & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & -b+4 & 2 & 0 \\ 0 & 0 & b & 3 & 2 \\ 0 & 0 & 0 & a & 2 \end{pmatrix}$$

$$\overset{f_{43}\left(\frac{-b}{-b+4}\right)}{\sim} \begin{pmatrix} a & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & -b+4 & 2 & 0 \\ 0 & 0 & 0 & \frac{-2b}{-b+4}+3 & 2 \\ 0 & 0 & 0 & a & 2 \end{pmatrix} \overset{f_{54}\left(\frac{(-b+4)(-a)}{12-5b}\right)}{\sim} \begin{pmatrix} a & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 4-b & 2 & 0 \\ 0 & 0 & 0 & \frac{-2b}{-b+4}+3 & 2 \\ 0 & 0 & 0 & 0 & \frac{2(-a)(-b+4)}{12-5b}+2 \end{pmatrix}$$

Para rango 4: $\frac{-2a(-b+4)}{12-5b}+2 = 0 \quad \wedge \quad (a)(1)(4-b)\left(\frac{-2b}{-b+4}+3\right) \neq 0$

$2ab - 8a = -24 + 10b \quad \wedge \quad (4a-ab)\left(\frac{-2b}{-b+4}+3\right) \neq 0$

$b \neq 4 \vee b \neq 12$

20) valores de b y a que vuelvan la matriz a $r(AB)=5 \vee r(AB)=4$

$r(AB)=3 \vee r(AB)=2$

$r(AB)=1$

$$A=\begin{bmatrix} a & 2 & 0 & 0 & 0 \\ a & 3 & 2 & 0 & 0 \\ a & b & b & 2 & 0 \\ 0 & 0 & b & 3 & 2 \\ 0 & 0 & 0 & a & 2 \end{bmatrix}; \quad B=\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix},$$

$-(2b-4)=-2b+4$

$$|B|=\begin{vmatrix} 2 & -1 & 0 & 0 & 0 \\ 0 & 3/2 & -3/2 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}, \quad |B|=3\neq 0; \rightarrow r(B)=5$$

$$|A| \underset{f_{31}(-1)}{\overset{f_{21}(-1)}{=}} \begin{vmatrix} a & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & b-2 & b & 2 & 0 \\ 0 & 0 & b & 3 & 2 \\ 0 & 0 & 0 & a & 2 \end{vmatrix} \underset{f_{32}(-(b-2))}{=} \begin{vmatrix} a & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 4-b & 2 & 0 \\ 0 & 0 & b & 3 & 2 \\ 0 & 0 & 0 & a & 2 \end{vmatrix}$$

$$\underset{f_{43}\left(\frac{-b}{4-b}\right)}{=} \begin{vmatrix} a & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 4-b & 2 & 0 \\ 0 & 0 & 0 & \frac{12-5b}{4-b} & 2 \\ 0 & 0 & 0 & a & 2 \end{vmatrix} \underset{f_{54}\left(-a\left(\frac{4-b}{12-5b}\right)\right)}{=} \begin{vmatrix} a & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 4-b & 2 & 0 \\ 0 & 0 & 0 & \frac{12-5b}{4-b} & 2 \\ 0 & 0 & 0 & 0 & m \end{vmatrix}$$

$$m = 2 - a\left(\frac{4-b}{12-5b}\right) = \frac{24-10b-4a+ab}{12-5b}$$

6) i) $A=\begin{bmatrix} 1 & 1 & 0 & 1 \\ 3 & 2 & -1 & 3 \\ m & 3 & -2 & 0 \\ -1 & 0 & -4 & 3 \end{bmatrix}$ ; $r(A)=4 \leftrightarrow |A|\neq 0 \rightarrow |A| = 1\cdot(-1)^2 \begin{vmatrix} 2 & -1 & 3 \\ 3 & -2 & 0 \\ 0 & -4 & 3 \end{vmatrix}$

$+ 1(-1)^3 \begin{vmatrix} 3 & -1 & 3 \\ m & -2 & 0 \\ -1 & -4 & 3 \end{vmatrix} + 0 + 1(-1)^{4+1} \begin{vmatrix} 3 & 2 & -1 \\ m & 3 & -2 \\ -1 & 0 & -4 \end{vmatrix}$

$|A| = (-12 \overset{39}{-36} + 9) + (-1)(\overset{-24-9m}{-18-12m-6+3m}) + (-1)(\overset{-35+8m}{-36+4-3+8m})$ ,

$|A| = 99+m \neq 0$ , $*\ r(A)=4 \Leftrightarrow m\neq -99$

$*\ r(A)=3 \Leftrightarrow r(A)\neq 4$ , $3\leq r(A)\leq 4$ , $r(A)\in \mathbb{Z}^+$

ii) $A=\begin{bmatrix} m & 1 & 1 \\ 1 & m & 1 \\ n & n & mn \end{bmatrix}$ ; $r(A)=3 \leftrightarrow |A|\neq 0 \rightarrow |A| = m^3 n + 2n - 3nm \neq 0$

$n\neq 0$

$m^3 - 3m + 2 \neq 0$

$(m-1)^2(m+2)\neq 0$

$m\neq 1 \vee m\neq -2$

$*\ r(A)=3 \Leftrightarrow \cdot\, m \in \mathbb{R}-\{1,-2\}$
$\cdot\, n\neq 0$

$*\ r(A) = 2 \Leftrightarrow * \begin{vmatrix} m & 1 \\ n & mn \end{vmatrix} \neq 0$ $\cdot\, n\neq 0$ $m\neq -1;$ $\cdot\, m\neq 1$ $\quad * \begin{vmatrix} 1 & 1 \\ n & mn \end{vmatrix} \neq 0$ $\cdot\, n\neq 0$ $\cdot\, m\neq 1$

$m^2-1=0$

$* \begin{vmatrix} 1 & m \\ n & n \end{vmatrix} \neq 0$ $\cdot\, n\neq 0$ $\cdot\, m\neq 1$ ; $* \begin{vmatrix} 1 & 1 \\ n & mn \end{vmatrix} \neq 0$ $\cdot\, n\neq 0$ $\cdot\, m\neq 1$ $\vee * \begin{vmatrix} m & 1 \\ n & mn \end{vmatrix} \neq 0$ $\cdot\, n\neq 0$ $m\neq 1; -1$

$m^2-1$

$* \begin{vmatrix} m & 1 \\ n & n \end{vmatrix} \neq 0$ $\cdot\, n\neq 0$ $\cdot\, m\neq 1$ $\vee * \begin{vmatrix} 1 & 1 \\ m & 1 \end{vmatrix} \neq 0$ $\cdot\, m\neq 1$ , $* \begin{vmatrix} m & 1 \\ 1 & 1 \end{vmatrix} \neq 0$ $\cdot\, m\neq -1$ , $* \begin{vmatrix} m & 1 \\ 1 & m \end{vmatrix} \neq 0$

$\cdot\, m\neq 1; -1$

$*\ r(A)=2 \Leftrightarrow r(A)\neq 3$ , $2\leq r(A)\leq 3$ , $r(A)\in \mathbb{Z}^+$

7) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix} = A\cdot B$ , $M = \begin{bmatrix} 1+\lambda \end{bmatrix}$

$r(M)=1, \forall \lambda \in \mathbb{R}-\{-1\}$

si $\lambda = -1 \rightarrow r(A)=0$

$$\frac{\quad}{2} \quad x = \frac{3-\sqrt{17}}{2}$$

(13) Resolver y clasificar el siguiente sistema

$$\begin{cases} 5x_1 + 3x_2 - x_3 - 3x_4 = 6 \\ 4x_1 - 2x_2 + x_3 + 2x_4 = -6 \\ -6x_1 - 8x_2 + 3x_3 + 8x_4 = -18 \\ 3x_1 - 7x_2 + 3x_3 + 7x_4 = -18 \end{cases}$$

Expresión matricial:

$$\underbrace{\begin{bmatrix} 5 & 3 & -1 & -3 \\ 4 & -2 & 1 & 2 \\ -6 & -8 & 3 & 8 \\ 3 & -7 & 3 & 7 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 6 \\ -6 \\ -18 \\ -18 \end{bmatrix}}_{b}$$

$$Aa = \left[\begin{array}{cccc|c} 5 & 3 & -1 & -3 & 6 \\ 4 & -2 & 1 & 2 & -6 \\ -6 & -8 & 3 & 8 & -18 \\ 3 & -7 & 3 & 7 & -18 \end{array}\right] \xrightarrow{f_{12}(-1)} \left[\begin{array}{ccccc} 1 & 5 & -2 & -5 & 12 \\ 4 & -2 & 1 & 2 & -6 \\ -6 & -8 & 3 & 8 & -18 \\ 3 & -7 & 3 & 7 & -18 \end{array}\right] \xrightarrow[f_{41}(-3)]{f_{21}(-4),\ f_{31}(6)}$$

$$\left[\begin{array}{cccc|c} 1 & 5 & -2 & -5 & 12 \\ 0 & -22 & 9 & 22 & -54 \\ 0 & 22 & -9 & -22 & 54 \\ 0 & -22 & 9 & 22 & -54 \end{array}\right] \xrightarrow[f_{32}(1)]{f_{42}(-1)} \left[\begin{array}{cccc|c} 1 & 5 & -2 & -5 & 12 \\ 0 & -22 & 9 & 22 & -54 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{f_2(-1/22)} \underbrace{\left[\begin{array}{cccc|c} 1 & 5 & -2 & -5 & 12 \\ 0 & 1 & -9/22 & -1 & 27/11 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]}_{Aa}$$

$$r(A) = r(Aa) = 2 = r < n = 4$$

$$\left.\begin{array}{l} x_1 + 5x_2 - 2x_3 - 5x_4 = 12 \\ x_2 - \frac{9}{22}x_3 - x_4 = 27/11 \end{array}\right\} \begin{array}{l} 2 \text{ ecuaciones} \\ 4 \text{ variables} \end{array} \quad \exists \text{ infinitas soluciones}$$

Sistema de ecuaciones lineales consistente de solución múltiple

15) Para que valores de $\lambda$ el sistema tiene infinitas soluciones

$$\begin{cases} (1-\lambda)x + y + z = 0 \\ 2x + (1-\lambda)y + 2z = 0 \\ x + y + (1-\lambda)z = 0 \end{cases}$$

Para que el sistema de ecuaciones tenga infinitas soluciones el rango debe ser menor que el número de incógnitas

#. incógnitas = 3

$\rightarrow R(A) < 3$

$$\rightarrow |A| = \begin{vmatrix} (1-\lambda) & 1 & 1 \\ 2 & (1-\lambda) & 2 \\ 1 & 1 & (1-\lambda) \end{vmatrix} = 0$$

$$\begin{vmatrix} (1-\lambda) & 1 & 1 \\ 2 & (1-\lambda) & 2 \\ 1 & 1 & (1-\lambda) \end{vmatrix} \xrightarrow{C_{13}(-1)} \begin{vmatrix} -\lambda & 1 & 1 \\ 0 & (1-\lambda) & 2 \\ \lambda & 1 & (1-\lambda) \end{vmatrix} \xrightarrow{F_{13}(1)} \begin{vmatrix} 0 & 2 & (2-\lambda) \\ 0 & (1-\lambda) & 2 \\ \lambda & 1 & (1-\lambda) \end{vmatrix}$$

$$= (-1)^{3+1} \lambda \left[4 - (2-\lambda)(1-\lambda)\right] = 0$$

$$\lambda(\lambda^2 - 3\lambda - 2) = 0$$

$\rightarrow \lambda = 0$ $\quad\rightarrow \lambda = \dfrac{3+\sqrt{17}}{2} \vee \lambda = \dfrac{3-\sqrt{17}}{2}$

**20)**

$$\left|\begin{array}{cccc|c} t & 1 & t & 1 & t \\ 2t & 1 & t & & t^2 \\ 1 & 1 & 2t & 1 & 0 \end{array}\right| \quad \left|\begin{array}{ccc|c} 1 & -1 & 2t-1 & 0 \\ -2t & 1 & -t & t^2 \\ t+1 & t & 1 & t \end{array}\right| \quad \left|\begin{array}{ccc|c} 1 & -1 & 2t-1 & 0 \\ 0 & -2t+1 & 4t^2-3t & t^2 \\ -t+1 & t & 1 & t \end{array}\right|$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2t-1 & 0 \\ 0 & -2t+1 & 4t^2-3t & t^2 \\ 0 & 1 & 2t^2-3t+2 & \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & -1 & 2t-1 & 0 \\ 0 & 1 & 2t^2-3t+2 & t \\ 0 & -2t+1 & 4t^2-3t & t^2 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & -1 & 2t-1 & 0 \\ 0 & 1 & 2t^2-3t+2 & t \\ 0 & 0 & 4t^3-4t^2+4t-2 & 3t^2-t \end{array}\right]$$

$x - y + (2t-1)z = 0$

$y + (2t^2-3t+2)z = t$ $\qquad 2t^3-2t^2+2t-1 \neq 0$

$(4t^3-4t^2+4t-2)z = 3t^2-t$

$\hookrightarrow$ Si $=0$ No existe solución

**21)** $\left[\begin{array}{ccc|c} 1 & -2 & 1 & a \\ 2 & 1 & 1 & b \\ 0 & 5 & -1 & c \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & -2 & 1 & a \\ 0 & 5 & -1 & -2a+b \\ 0 & 5 & -1 & c \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & -2 & 1 & a \\ 0 & 5 & -1 & -2a+b \\ 0 & 0 & 0 & 2a-b+c \end{array}\right]$

1) $2a-b+c \neq 0$

$\left[\begin{array}{ccc|c} 1 & -2 & +1 & a \\ 0 & 5 & -1 & -2a+b \\ 0 & 0 & 0 & 2a-b+c \end{array}\right]$ $\qquad x - 2y + z = a$, $\quad 5y - z = -2a+b$, $\quad 0 = 2a-b+c$ $\qquad$ No existe solución

2) $2a-b+c=0$

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & a \\ 0 & 5 & -1 & -2a+b \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\qquad x - 2y + z = a$, $\quad 5y - z = -2a+b$

$x = \frac{a+2b}{5} - \frac{3}{5}z$

$y = \frac{-2a+b}{5} + \frac{1}{5}z$

$z = z$

# Universidad nacional de ingeniería

**Facultad de ingeniería económica, estadística y ciencias sociales**

**Escuela profesional de ingeniería económica**

## Segundo Trabajo

## Curso: Álgebra Lineal

## Profesor: Mejia



## Alumno: Saucedo Batallanos Marlon Nilo